

METHODS FOR CALCULATING THE NAVIGATION INFORMATION
FOR "MARS" SPACECRAFT

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METHODS FOR CALCULATING THE NAVIGATION INFORMATION¹ FOR "MARS" SPACECRAFT

P.E. El'yasberg, E.P. Aleksashin, N.G. Khavenson

The article discusses methods used to calculate navigation ^{/2*} parameters for interplanetary "Mars" spacecraft. The methods are based on the numerical integration of the differential equations of motion of the spacecraft. To calculate the position of celestial bodies, Newcomb expansions are used in the case of planets and polynomials based on Brown's theory in the case of the moon. The methods can be used for navigational calculations of any interplanetary spacecraft in the Earth-Mars or Earth-Moon space.

I. General Comments

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Navigation information includes two types of parameters: general navigation information which characterizes the positions and velocities of celestial bodies and the spacecraft (SC) at given instants of time t_i (year, date, time) in specially selected coordinate systems, and instrumental navigation information which describes the orientation of the characteristic axes of the instrument in the same or different coordinate systems. The choice of the coordinate system depends on the vector of the SC trajectory. In the problem of calculating the navigation information for the "Mars" spacecraft two sectors are considered: the Earth-Mars flight trajectory [1-4] and the orbit of an artificial Mars satellite. The navigation information is calculated at given instants of time t_i by integrating numerically the differential equations describing the motion of the spacecraft, using the corresponding Newcomb expansions describing the evolution of the mean orbital elements of the celestial bodies. The initial conditions for the numerical integration are estimates of the orbital parameters of the spacecraft obtained from observation data.

*Numbers in the margin indicate pagination in the foreign text.

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II. Methods for Calculating the Navigation Information on the Earth-Mars Trajectory /4

1. Description of the Motion of the Spacecraft

The motion of the "Mars" spacecraft on the flight trajectory  is considered in a system of rectangular geocentric geoequatorial coordinates at the 1960.0 epoch (the mean equator and Earth equinox at the 1960 epoch $Jd. = 2\,436\,934.8453$). This system is referred to in [1] as the absolute system. The differential equations of the spacecraft which take into account perturbing accelerations due to the non-centered gravitational field $\oplus$, light radiation, attraction, $\odot$, and ψ are written in the form

$$\begin{aligned} \dot{x}_{\oplus\Delta} &= v_{x\oplus\Delta}; \\ \dot{y}_{\oplus\Delta} &= v_{y\oplus\Delta}; \\ \dot{z}_{\oplus\Delta} &= v_{z\oplus\Delta}; \\ \dot{v}_{x\oplus\Delta} &= -\frac{\mu_{\oplus} x_{\oplus\Delta}}{r_{\oplus\Delta}^3} - \frac{3/2 (\mu_{\oplus} R_{\oplus}^2 C_{20\oplus})}{r_{\oplus\Delta}^4} \left(\frac{x_{\oplus\Delta}}{r_{\oplus\Delta}} \right) \left[5 \left(\frac{z_{\oplus\Delta}}{r_{\oplus\Delta}} \right)^2 - 1 \right] + \\ &\quad + \sum_{j=1}^n \mu_j \left(\frac{x_{\oplus j} - x_{\oplus\Delta}}{r_{j\Delta}^3} - \frac{x_{\oplus j}}{r_{j\Delta}^3} \right) - \frac{x_{\oplus\Delta}}{r_{\oplus\Delta}^2} \cdot \frac{x_{\oplus\odot} - x_{\oplus\Delta}}{r_{\oplus\Delta}}; \\ \dot{v}_{y\oplus\Delta} &= -\frac{\mu_{\oplus} y_{\oplus\Delta}}{r_{\oplus\Delta}^3} - \frac{3/2 (\mu_{\oplus} R_{\oplus}^2 C_{20\oplus})}{r_{\oplus\Delta}^4} \left(\frac{y_{\oplus\Delta}}{r_{\oplus\Delta}} \right) \left[5 \left(\frac{z_{\oplus\Delta}}{r_{\oplus\Delta}} \right)^2 - 1 \right] + \\ &\quad + \sum_{j=1}^n \mu_j \left(\frac{y_{\oplus j} - y_{\oplus\Delta}}{r_{j\Delta}^3} - \frac{y_{\oplus j}}{r_{j\Delta}^3} \right) - \frac{y_{\oplus\Delta}}{r_{\oplus\Delta}^2} \cdot \frac{y_{\oplus\odot} - y_{\oplus\Delta}}{r_{\oplus\Delta}}; \\ \dot{v}_{z\oplus\Delta} &= -\frac{\mu_{\oplus} z_{\oplus\Delta}}{r_{\oplus\Delta}^3} - \frac{3/2 (\mu_{\oplus} R_{\oplus}^2 C_{20\oplus})}{r_{\oplus\Delta}^4} \left(\frac{z_{\oplus\Delta}}{r_{\oplus\Delta}} \right) \left[5 \left(\frac{z_{\oplus\Delta}}{r_{\oplus\Delta}} \right)^2 - 3 \right] + \\ &\quad + \sum_{j=1}^n \mu_j \left(\frac{z_{\oplus j} - z_{\oplus\Delta}}{r_{j\Delta}^3} - \frac{z_{\oplus j}}{r_{j\Delta}^3} \right) - \frac{z_{\oplus\Delta}}{r_{\oplus\Delta}^2} \cdot \frac{z_{\oplus\odot} - z_{\oplus\Delta}}{r_{\oplus\Delta}}. \end{aligned} \quad (1)$$

where

$$\bar{R}_{\Delta}^T = \|x_{\Delta} \ y_{\Delta} \ z_{\Delta}\|$$

is the geocentric radius-vector of the spacecraft,

$$\bar{V}_{\Delta}^T = \|v_{\Delta} \ y_{\Delta} \ z_{\Delta}\|$$

is the geocentric velocity of the spacecraft,

$$\bar{R}_{\Theta j}^T = \|x_{\Theta j} \ y_{\Theta j} \ z_{\Theta j}\|$$

is the geocentric radius-vector of the j-th celestial body,

$$r_{\Delta} = |\bar{R}_{\Delta}| = \sqrt{x_{\Delta}^2 + y_{\Delta}^2 + z_{\Delta}^2},$$

$$r_{\Theta j} = |\bar{R}_{\Theta j}| = \sqrt{x_{\Theta j}^2 + y_{\Theta j}^2 + z_{\Theta j}^2},$$

is the radius vector of the spacecraft relative to the center of mass of the perturbing body,

$$\bar{R}_{j\Delta} = \bar{R}_{\Delta} - \bar{R}_{\Theta j}$$

$$r_{j\Delta} = |\bar{R}_{j\Delta}| = \sqrt{(x_{\Theta j} - x_{\Delta})^2 + (y_{\Theta j} - y_{\Delta})^2 + (z_{\Theta j} - z_{\Delta})^2},$$

$$\Delta, \omega, \delta, \sigma, \psi$$

are respectively the spacecraft, the Earth, the Moon, the Sun, Mars, and Jupiter.

μ_j is the gravitational parameter of the j-th perturbing body,

$\mu_{\oplus}$ is the gravitational parameter of Earth,

$R_{\oplus}$ is the Earth's equatorial radius,

C_{200} is the coefficient of the zonal harmonic in the series expansion for the force function of Earth's gravitation in spherical coordinates,

χ_{Δ} is the light radiation coefficient of the spacecraft.¹

The initial conditions for the integration of system (1) are the coordinates and velocity components of the spacecraft in an absolute geocentric coordinate system which were obtained from estimates of the orbital parameters based on the results of trajectory measurements. Usually these are given in the form:

t_0 (year, date, time) the instant, Moscow daylight saving time, to which the initial conditions are referred;

$\bar{R}_{\Delta 0} = \begin{pmatrix} x_{\Delta 0} \\ y_{\Delta 0} \\ z_{\Delta 0} \end{pmatrix}$ the initial geocentric radius-vector;

¹The values of all trajectory constants are given in Section V. Their numerical values which were used in the calculation may vary to the extent that they are made more precise on the basis of an analysis of trajectory measurements.

$$\vec{V}_{000} = \begin{pmatrix} V_{x_{000}} \\ V_{y_{000}} \\ V_{z_{000}} \end{pmatrix}$$
 the initial geocentric velocity vector of the spacecraft.

During the calculations, the initial conditions are given, as a rule, once per week.

2. Calculation of the Positions of Celestial Bodies

Since the motion of the spacecraft is considered in a geocentric geoequatorial coordinate system at the 1960.0 epoch, the positions of the heavenly bodies must also be given in this coordinate system. Newcomb's theory of motion with Ross' corrections is used for the calculation of the positions of large planets in the case of Mars. The calculation of the positions of the Moon is based on Brown's theory, taking into consideration the recommendations which were made at the 8th and 13th general session of the International Astronomical Union.

The mean orbital elements of celestial bodies which are obtained on the basis of these theories are referred to a heliocentric ecliptic coordinate system (the mean ecliptic and Earth equinox at the current instant). We introduce the following ecliptic orbital elements for the celestial body:

- L the mean longitude,
- π the perihelion longitude,
- Ω the ascending node longitude,
- i the inclination of the orbital plane,
- e the eccentricity,
- a the major semiaxis.

Then the mean Earth orbital elements will be determined from the Newcomb formulas [2]:

$$\left. \begin{aligned}
 L_{\odot} &= 99^{\circ} 41' 48''.04 + 129602768''.15 T_A + 1''.089 T_A^2, \\
 \pi_{\odot} &= 101^{\circ} 15' 15''.0 + 6189''.03 T_A + 1''.63 T_A^2 + 0''.012 T_A^3, \\
 \Omega_{\odot} &= 0.
 \end{aligned} \right\} \quad (2)$$

$$\begin{aligned} \ell_{\odot\oplus} &= 0.01675104 - 0.00004180 T_{\Delta} - 0.000000126 T_{\Delta}^2, \\ \iota_{\odot\oplus} &= 0, \\ u_{\odot\oplus} &= 1.00000017. \end{aligned}$$

The mean orbital elements of Mars will be determined from the Newcomb formulas with Ross' corrections [2]:

$$\begin{aligned} L_{\odot\odot} &= 293^{\circ}44'51''.46 + 68910117''.33 T_{\Delta} + 1''.1184 T_{\Delta}^2, \\ \Pi_{\odot\odot} &= 334^{\circ}13'05''.53 + 6626''.73 T_{\Delta} + 0''.4675 T_{\Delta} - 0''.0043 T_{\Delta}^3, \\ \Omega_{\odot\odot} &= 48^{\circ}47'11''.19 + 2775''.57 T_{\Delta} - 0''.005 T_{\Delta} - 0''.0192 T_{\Delta}^2, \\ \ell_{\odot\odot} &= 0.09331290 + 0.000092064 T_{\Delta} - 0.000000077 T_{\Delta}^2, \\ \iota_{\odot\odot} &= 1^{\circ}51'01''.20 - 2''.430 T_{\Delta} + 0.0454 T_{\Delta}^2, \\ q_{\odot\odot} &= 1.52368839. \end{aligned} \tag{3}$$

The mean orbital elements of Jupiter will be determined from Newcomb's formulas [2]:

$$\begin{aligned} L_{\odot\vee} &= 238^{\circ}02'57''.32 + 10930687''.148 T_{\Delta} + 1''.20486 T_{\Delta}^2 - 0''.005936 T_{\Delta}^3, \\ \Pi_{\odot\vee} &= 12^{\circ}43'15''.34 + 5795''.862 T_{\Delta} + 3''.80258 T_{\Delta}^2 - 0''.01236 T_{\Delta}^3, \\ \Omega_{\odot\vee} &= 99^{\circ}26'36''.19 + 3637''.908 T_{\Delta} + 1''.2680 T_{\Delta}^2 - 0''.03064 T_{\Delta}^3, \\ \ell_{\odot\vee} &= 0.04833475 + 0.000164180 T_{\Delta} - 0.0000004676 T_{\Delta}^2 - 0.0000000177 T_{\Delta}^3, \\ \iota_{\odot\vee} &= 1^{\circ}18'31''.45 - 20''.506 T_{\Delta} + 0''.014 T_{\Delta}^2, \\ q_{\odot\vee} &= 5.202561. \end{aligned} \tag{4}$$

The mean orbital elements of the Moon are referred to a geocentric ecliptic coordinate system with a mean ecliptic and Earth equinox at the current instant. Let us introduce:

- α the mean longitude of the Moon,
- ℓ the mean anomaly of the Moon,
- ℓ the mean anomaly of the Sun,
- F the mean argument of latitude of the Moon,
- D the difference between the mean latitudes of the Moon and Sun,
- π the lunar parallax.

Then the mean elements which were introduced are defined by the expressions [2]:

$$\begin{aligned}
 \ell &= 270^{\circ} 26' 02'' .99 + 481267^{\circ} 52' 59'' .31 T_{\Delta} - 4'' .08 T_{\Delta}^2 + 0'' .0068 T_{\Delta}^3, \\
 \ell' &= 296^{\circ} 06' 16'' .59 + 477198^{\circ} 50' 56'' .79 T_{\Delta} + 37'' .09 T_{\Delta}^2 + 0'' .0518 T_{\Delta}^3, \\
 \ell'' &= 358^{\circ} 28' 33'' .0 + 35999^{\circ} 02' 59'' .10 T_{\Delta} - 0'' .54 T_{\Delta}^2 - 0'' .0120 T_{\Delta}^3, \\
 F &= 11^{\circ} 15' 03'' .20 + 483202^{\circ} 01' 30'' .54 T_{\Delta} - 11'' .56 T_{\Delta}^2 - 0'' .0012 T_{\Delta}^3, \\
 D &= 350^{\circ} 44' 14'' .95 + 445267^{\circ} 06' 51'' .18 T_{\Delta} - 5'' .17 T_{\Delta}^2 + 0'' .0068 T_{\Delta}^3.
 \end{aligned} \tag{5}$$

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The ecliptic right ascension, declination, and the horizontal lunar parallax will be determined from the expansions:

$$\begin{aligned}
 \alpha_e &= \alpha + 22639.580 \sin \ell - 4586.438 \sin (\ell - 2D) + \\
 &+ 2369.899 \sin (2D) + 769.021 \sin (2\ell) - \\
 &- 668.944 \sin \ell' - 411.614 \sin (2F) - \\
 &- 211.658 \sin (2\ell - 2D) - 206.219 \sin (\ell + \ell' - 2D) + \\
 &+ 191.954 \sin (\ell + 2D) - 165.351 \sin (\ell' - 2D) + \\
 &+ 147.878 \sin (\ell - \ell') - 124.785 \sin D - \\
 &- 109.804 \sin (\ell + \ell') - 55.174 \sin (2F - 2D) - \\
 &- 45.100 \sin (\ell + 2F) + 39.532 \sin (\ell - 2F) - \\
 &- 38.428 \sin (\ell - 4D) + 36.124 \sin (3\ell) - \\
 &- 30.775 \sin (2\ell - 4D) - 28.511 \sin (\ell - \ell' - 2D) - \\
 &- 24.451 \sin (\ell' + 2D); \\
 \delta_e &= 18461.480 \sin F + 1010.180 \sin (\ell - F) - \\
 &- 999.695 \sin (F - \ell) - 623.658 \sin (F - 2D) + \\
 &+ 199.485 \sin (F + 2D) - \ell - 166.577 \sin (\ell + F - 2D) + \\
 &+ 117.262 \sin (F + 2D) + 61.913 \sin (2\ell + F) - \\
 &- 33.359 \sin (F - 2D - \ell) - \\
 &- 31.763 \sin (F - 2\ell) - 29.689 \sin (\ell' + F - 2D);
 \end{aligned} \tag{6}$$

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$$\begin{aligned}
\pi_g = & \pi_0 + 186,5398 \cos \ell + 34,3117 \cos (\ell - 2D) + \\
& + 28,2333 \cos (2D) + 10,1657 \cos (2\ell) + \\
& + 3,0861 \cos (\ell + 2D) + 1,9202 \cos (\ell' - 2D) + \\
& + 1,4455 \cos (\ell + \ell' - 2D) + 1,1542 \cos (\ell - \ell') - \\
& - 0,9752 \cos D - 0,9502 \cos (\ell + \ell') - \\
& - 0,7136 \cos (\ell - 2F) + 0,0215 \cos 3\ell + \\
& + 0,6008 \cos (\ell - 4D),
\end{aligned}$$

where

$$T_\Delta = \frac{JD_t - 2415020.0}{36525},$$

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(7)

JD_t is the Julian day at the current instant.

Now the rectangular heliocentric geoequatorial coordinates of Earth, Mars and Jupiter will be determined from the formulas:

$$\begin{aligned}
\tilde{X}_{op} &= A_{xp} (\cos E_p - \ell_{op}) + B_{xp} \sin E_p, \\
\tilde{Y}_{op} &= A_{yp} (\cos E_p - \ell_{op}) + B_{yp} \sin E_p, \\
\tilde{Z}_{op} &= A_{zp} (\cos E_p - \ell_{op}) + B_{zp} \sin E_p,
\end{aligned} \tag{8}$$

where

$$\begin{aligned}
A_{xp} &= a_{op} P_x ; & B_{xp} &= a_{op} \sqrt{1 - \ell_{op}^2} Q_{xp}, \\
A_{yp} &= a_{op} P_y ; & B_{yp} &= a_{op} \sqrt{1 - \ell_{op}^2} Q_{yp}, \\
A_{zp} &= a_{op} P_z ; & B_{zp} &= a_{op} \sqrt{1 - \ell_{op}^2} Q_{zp}.
\end{aligned} \tag{9}$$

E_p is the eccentric anomaly of the planet which is determined by solving iteratively the Kepler equation with the zero approximation:

$$E_{op} = M_p + \ell_{op} \sin M_p + \frac{1}{2} \ell_{op}^2 \sin 2M_p$$

(10)

and successive approximations:

$$E_{np} = M_p + \ell_{op} \sin E_{n-1p},$$

(11)

where $M_p = L_{op} - \tilde{H}_{op}$ is the mean anomaly of the planet.

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$$\begin{aligned}
 P_{x_p} &= \cos \omega_p \cos \Omega_p - \sin \omega_p \sin \Omega_p \cos i_p ; \\
 P_{y_p} &= (\cos \omega_p \sin \Omega_p + \sin \omega_p \cos \Omega_p \cos i_p) \cos \xi - \\
 &\quad - \sin \omega_p \sin i_p \sin \xi ; \\
 P_{z_p} &= (\cos \omega_p \sin \Omega_p + \sin \omega_p \cos \Omega_p \cos i_p) \sin \xi + \\
 &\quad + \sin \omega_p \sin i_p \cos \xi ; \\
 Q_{x_p} &= -\sin \omega_p \cos \Omega_p - \cos \omega_p \sin \Omega_p \cos i_p ; \\
 Q_{y_p} &= (-\sin \omega_p \sin \Omega_p + \cos \omega_p \cos \Omega_p \cos i_p) \cos \xi - \cos \omega_p \sin i_p \sin \xi ; \\
 Q_{z_p} &= (-\sin \omega_p \sin \Omega_p + \cos \omega_p \cos \Omega_p \cos i_p) \sin \xi + \cos \omega_p \sin i_p \cos \xi ;
 \end{aligned} \tag{12}$$

$\omega_p = \tilde{H}_{op} - \Omega_{op}$ is the ascending node perihelion of the planet.

$$\xi = 23.45229444 - 0.13012500 \cdot 10^{-1} T_A - 0.16388889 \cdot 10^{-5} T_A^2 + 0.50277778 \cdot 10^{-6} T_A^3 . \tag{13}$$

The rectangular geocentric geoequatorial coordinates of Mars, the Sun and Jupiter will be determined from the formulas which translate the coordinates to a new origin. We have:

$$\begin{aligned}
 \tilde{X}_{op} &= \tilde{X}_{op} - \tilde{X}_{os} , \\
 \tilde{Y}_{op} &= \tilde{Y}_{op} - \tilde{Y}_{os} , \\
 \tilde{Z}_{op} &= \tilde{Z}_{op} - \tilde{Z}_{os} .
 \end{aligned} \tag{14}$$

The rectangular geocentric geoequatorial coordinates of the Moon will be determined from the formulas:

$$\begin{pmatrix} \tilde{X}_{+1} \\ \tilde{Y}_{+1} \\ \tilde{Z}_{+1} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \xi & -\sin \xi \\ 0 & \sin \xi & \cos \xi \end{pmatrix} \cdot \begin{pmatrix} r_d \cos \delta_k \cos \ell_k \\ r_d \cos \delta_k \sin \ell_k \\ r_d \sin \delta_k \end{pmatrix} \tag{15}$$

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where $r_d = \frac{R_{dm}}{\sin h_k}$ is the distance between the Earth and the Moon.

Since the coordinates of Mars, the Sun, Jupiter, and the Moon which were obtained from formulas (8), (13), (15) are referred to

the mean equator and equinox of the Earth at the current instant, and the navigation parameters are calculated in the 1960.0 system, precession must be taken into account. We introduce the precession matrix P which is the matrix of the transformation which transforms the coordinates "mean equator and Earth equinox at the current instant" to the mean equator and Earth equinox at the standard 1960.0 calculated epoch." The elements of the matrix P will be determined from the formulas:

$$\left. \begin{aligned} P_{11} &= -\sin \chi \sin \omega + \cos \chi \cos \omega \cos \psi; & P_{12} &= \sin \chi \cos \omega + \cos \chi \sin \omega \cos \psi; \\ P_{13} &= \cos \chi \sin \psi; \\ P_{21} &= -\cos \chi \sin \omega - \sin \chi \cos \omega \cos \psi; & P_{22} &= \cos \chi \cos \omega - \sin \chi \sin \omega \cos \psi; \\ P_{23} &= -\sin \chi \sin \psi; \\ P_{31} &= -\cos \omega \sin \psi; & P_{32} &= -\sin \omega \sin \psi; & P_{33} &= \cos \psi. \end{aligned} \right\} \quad (16)$$

where χ, ω, ψ are the Newcomb angles

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$$\left. \begin{aligned} \chi &= (25042''.53 + 139''.73\tau + 0''.06\tau^2)t + (30''.23 - 0''.27\tau)t^2 + 18''.00t^3; \\ \omega &= \chi + (79''.27 + 0''.66\tau)t^2 + 0''.32t^3; \\ \psi &= (20046''.85 - 85''.33\tau - 0''.37\tau^2)t + (-42''.67 - 0''.37\tau)t^2 - 41''.80t^3; \\ \tau &= \frac{JD_{1960.0} - 2415020.0}{365242.19871}; \\ t &= \frac{JD_1 - JD_{1960.0}}{365242.19871}. \end{aligned} \right\} \quad (17)$$

Now the coordinates of the celestial bodies in the rectangular geocentric geoequatorial system (mean equator and Earth equinox at the 1960.0 epoch) are determined from the expressions:

$$\left\| \begin{array}{c} X_{\phi j} \\ Y_{\phi j} \\ Z_{\phi j} \end{array} \right\| = P \cdot \left\| \begin{array}{c} \tilde{X}_{\phi j} \\ \tilde{Y}_{\phi j} \\ \tilde{Z}_{\phi j} \end{array} \right\| \quad (18)$$

These coordinates are included in the right members of Eqs. (1), and are also the initial coordinates used in the calculation of any navigation data for the spacecraft on the Earth-Mars flight trajectory.

III. Method for Calculating the Navigation Information on the Orbit of an Artificial Mars Satellite /15

1. Description of the Motion of the Spacecraft

The motion of the "Mars" spacecraft in an orbit around Mars will be considered in a rectangular areocentric, areo-equatorial coordinate system, with the mean equator and Mars equinox at the initial calculated epoch which is given near the instant when the spacecraft enters the artificial satellite orbit. The initial calculated epoch T_0 is defined by the year, date and the instant in universal time UTO.

The differential equations of motion of the spacecraft which take into account the perturbing accelerations due to the non-centered gravitational field of Mars, solar and Jupiter gravitation, and light radiation are analogous to Eqs. (1) with the difference that the right members contain quantities with the symbol $\oplus$ instead of $\odot$ and instead of $\sum_{j=\odot, \oplus, \dots}$ we write $\sum_{j=\odot, \oplus}$, which describes the perturbations from the Sun and Jupiter. We also note that the term in the right members of (1) which describes the perturbing acceleration due to light radiation must only be taken into account in the case when the spacecraft is not in the shadow of Mars. When (1) is integrated, the condition for the illumination of the spacecraft by the Sun must be checked. If z_0 is the zenith distance of the Sun at the point of the spacecraft, the illumination condition is defined by the inequalities:

$$z_0 \begin{cases} > z_{0 \max} \text{ , SC in shadow} \\ < z_{0 \max} \text{ , SC illuminated} \end{cases} \quad (19)$$

where

$$Z_{\text{max}} = \pi - \arcsin\left(\frac{R_{30}}{Z_{\Delta 0}}\right), \quad (20)$$

$$Z_0 = \frac{\pi}{2} - \arcsin\left(\frac{X_{\Delta 0} X_{\Delta 0} + Y_{\Delta 0} Y_{\Delta 0} + Z_{\Delta 0} Z_{\Delta 0}}{Z_{\Delta 0}^2} - \frac{Z_{\Delta 0}}{Z_{\Delta 0}}\right), \quad (21)$$

The initial conditions for the integration are the coordinates /16 and the velocity components of the spacecraft in the coordinate system which was described above.

2. Calculation of the Positions of Celestial Bodies

To calculate the position of the Sun, Jupiter and Earth in a rectangular areoequatorial system with mean equator and Mars equinox at the initial calculated epoch T_0 , we need the transformation matrix $P_{\text{e-}j}$ which transforms the coordinates mean equator and Earth equinox at the epoch T_0 to the mean equator and Mars equinox at the epoch T_0 . The elements of this matrix will be determined from the formulas:

$$\begin{aligned} P_{\text{e-}11} &= \cos \omega_j \cos \Omega_j - \sin \omega_j \sin \Omega_j \cos i_j; \\ P_{\text{e-}12} &= \cos \omega_j \sin \Omega_j + \sin \omega_j \cos \Omega_j \cos i_j; \\ P_{\text{e-}13} &= \sin \omega_j \sin i_j; \\ P_{\text{e-}21} &= -\sin \omega_j \cos \Omega_j - \cos \omega_j \sin \Omega_j \cos i_j; \\ P_{\text{e-}22} &= -\sin \omega_j \sin \Omega_j + \cos \omega_j \cos \Omega_j \cos i_j; \\ P_{\text{e-}23} &= \cos \omega_j \sin i_j; \\ P_{\text{e-}31} &= \sin \Omega_j \sin i_j; \\ P_{\text{e-}32} &= -\sin i_j \cos \Omega_j; \\ P_{\text{e-}33} &= \cos i_j; \end{aligned} \quad (22)$$

where

$$\begin{aligned} \sin \omega_j &= \frac{-\sin \tilde{Z} \sin (\tilde{X} - i)}{\sin I}, \\ \cos \omega_j &= \frac{\cos (\tilde{X} - i) \cos (\tilde{Y} - \delta_0) - \sin (\tilde{X} - i) \sin (\tilde{Y} - \delta_0) \cos \tilde{Z}}{\sin I}, \end{aligned} \quad \text{/17}$$

$$\Omega_{\mathcal{A}} = \alpha_0 + \frac{\pi}{2},$$

$$i_{\mathcal{A}} = \frac{\pi}{2} - \delta_0,$$

$$\cos I = \cos(\tilde{\chi} - i) \sin(\tilde{\psi} - \delta_0) + \sin(\tilde{\chi} - i) \cos(\tilde{\psi} - \delta_0) \cos \tilde{z},$$

$$\sin I = \sqrt{1 - \cos^2 I},$$

$$\alpha_0 = 316.844 + 0.6533 T_1,$$

$$\delta_0 = +53.009 + 0.3542 T_1,$$

$$\cos \tilde{z} = \cos \tilde{\epsilon} \sin \Omega \cos \alpha_0 - \cos \Omega \sin \alpha_0,$$

$$\sin \tilde{z} = \sqrt{1 - \cos^2 \tilde{z}},$$

$$\sin \tilde{\chi} = \frac{\sin \tilde{\epsilon} \cos \alpha_0}{\sin \tilde{z}},$$

$$\cos \tilde{\chi} = \frac{-\cos \tilde{\epsilon} \cos \Omega \cos \alpha_0 - \sin \Omega \sin \alpha_0}{\sin \tilde{z}},$$

$$\sin \tilde{\psi} = \frac{\sin \tilde{\epsilon} \sin \Omega}{\sin \tilde{z}},$$

$$\cos \tilde{\psi} = \frac{\cos \tilde{\epsilon} \sin \Omega \sin \alpha_0 + \cos \Omega \cos \alpha_0}{\sin \tilde{z}},$$

$$T_1 = \frac{JD_{T_0} - JD_{1950.0}}{36525}.$$

Using formulas (2)-(4), (8)-(14), (16)-(18) with

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$$\left[\begin{array}{l} T_{\Delta} = \frac{(JD)_{T_0} - 2415020.0}{36525} ; \\ \tau = \frac{JD_{T_0} - 2415020.0}{365242.19871} ; \\ t = \frac{JD_t - JD_{T_0}}{365242.19871} \end{array} \right]$$

we obtain the rectangular geocentric geoequatorial coordinates of the Sun and Mars (the mean equator and Earth equinox at the initial calculated epoch T_0). Next, we translate the origin to the center of mass of Mars and multiply the coordinates which were obtained by the matrix $P_{\mathcal{A}}$.

$$\begin{aligned} \tilde{R}_{\mathcal{P}} &= \bar{R}_{\mathcal{O}} - \bar{R}_{\mathcal{O}\mathcal{P}}; \\ \bar{R}_{\mathcal{P}} &= P_{\mathcal{O}\mathcal{P}} \tilde{R}_{\mathcal{P}}. \end{aligned} \quad (24)$$

3. Calculation of the Positions of an Artificial Satellite of Mars in an Areographic Coordinate System

Often the results of scientific experiments which are carried out in artificial satellites must be referred to the surface of Mars. For this purpose, we introduce a rectangular areocentric coordinate system which is rigidly referred to the rotating Mars. The X axis will be directed along the 0-th meridian, the Z axis along the central rotation axis, and Y in such a way that the system is a right-handed system. Let $S_{\mathcal{O}}$ be the Martian sidereal time at the 0-th Mars meridian at the instant T_0 and $S_{\mathcal{P}}$ be the sidereal time at the current instant t . Then

$$S_{\mathcal{O}} = 149.479 + \tilde{\omega}_{\mathcal{P}} (JD_{T_0} - 2418322.0 + \Delta T^{(A)} - 0.000096), \quad (25)$$

where $\tilde{\omega}_{\mathcal{P}}$ is the angular velocity with which Mars rotates, JD_{T_0} is the Julian day at the initial calculated epoch, $\Delta T^{(A)}$ is the ephemeris correction,

and

$$S_{\mathcal{P}} = S_{\mathcal{O}} + \tilde{\omega}_{\mathcal{P}} (t - t_0). \quad (26)$$

The areocentric coordinates of the spacecraft and the celestial bodies in a moving coordinate system will be determined from the formulas:

$$\begin{pmatrix} x_{\mathcal{P}} \\ y_{\mathcal{P}} \\ z_{\mathcal{P}} \end{pmatrix} = \begin{pmatrix} \cos S_{\mathcal{P}} \sin S_{\mathcal{P}} & 0 \\ -\sin S_{\mathcal{P}} \cos S_{\mathcal{P}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_{\mathcal{P}} \\ y_{\mathcal{P}} \\ z_{\mathcal{P}} \end{pmatrix} \quad (27)$$

where p denotes not only the planet but also the spacecraft.

The areodesic latitude $B_{\mathcal{P}}$, longitude $L_{\mathcal{P}}$ and altitude $H_{\mathcal{P}}$ will be determined from the expressions:

$$\begin{aligned}
 B_{\mathcal{J}_p} &= \arcsin \frac{z_{\mathcal{J}_p}}{\sqrt{(1-\alpha_{\mathcal{J}})^4(x_{\mathcal{J}_p}^2 + y_{\mathcal{J}_p}^2) + z_{\mathcal{J}_p}^2}}; \\
 L_{\mathcal{J}_p} &= \arctg\left(\frac{y_{\mathcal{J}_p}}{x_{\mathcal{J}_p}}\right), \\
 H_{\mathcal{J}_p} &= z_{\mathcal{J}_p} R_{\mathcal{J}} \left[1 - \alpha_{\mathcal{J}} \left(\frac{z_{\mathcal{J}_p}}{z_{\mathcal{J}_p}} \right)^2 \right], \\
 z_{\mathcal{J}_p} &= \sqrt{x_{\mathcal{J}_p}^2 + y_{\mathcal{J}_p}^2 + z_{\mathcal{J}_p}^2},
 \end{aligned} \tag{28}$$

where

$R_{\mathcal{J}}$ is the equatorial radius of Mars;
 $\alpha_{\mathcal{J}}$ is the geometric oblateness of Mars.

IV. Method for Calculating the Instrumental Navigation Parameters /20

The transformation from the fixed coordinate systems $\{1, 2, 3\}$ to which the apparatus is referred to an areocentric system A is defined by the matrix A . To determine this matrix at each instant of time, we use the solar and Earth coordinates which were calculated in III.2. The directions of the Sun and Earth in the associated coordinate system $\{1, 2, 3\}$ are given in terms of the direction cosines of the axes of the antenna on Earth and the optical solar detector.

Assuming that the direction to the Sun and Earth from the satellite coincides with the corresponding directions from the center of Mars, we can determine the matrix A using the following formulas:

$$\begin{aligned}
 A &= \bar{M}_1 \bar{N}_1^T + \bar{M}_2 \bar{N}_2^T + \bar{M}_3 \bar{N}_3^T; \\
 \bar{M}_1 &= \frac{\bar{r}_{\mathcal{S}_0}}{|\bar{r}_{\mathcal{S}_0}|}, \quad \bar{M}_2 = \frac{\bar{r}_{\mathcal{S}_0} \times \bar{r}_{\mathcal{E}_0}}{|\bar{r}_{\mathcal{S}_0} \times \bar{r}_{\mathcal{E}_0}|}, \quad \bar{M}_3 = \bar{M}_1 \times \bar{M}_2, \\
 \bar{N}_1 &= \bar{p}_0, \quad \bar{N}_2 = \frac{\bar{p}_0 \times \bar{p}_0}{|\bar{p}_0 \times \bar{p}_0|}, \quad \bar{N}_3 = \bar{N}_1 \times \bar{N}_2,
 \end{aligned} \tag{29}$$

where $\bar{r}_{\mathcal{S}_0}, \bar{r}_{\mathcal{E}_0}$ are the areocentric radius-vectors of the Sun and Earth,

$\bar{p}_\odot^\circ, \bar{p}_\oplus^\circ$ are the unit vectors from the center of the object in the direction of the Sun and Earth and the symbol T denotes transposition.

Thus, any given direction $\bar{v}_i^\circ$ of the axis of the i-th instrument in the coordinate system ξ, η, ζ determines a direction $\bar{l}_i^\circ$ in the coordinate system $x_\delta, y_\delta, z_\delta$.

$$\bar{l}_i^\circ = A \bar{v}_i^\circ \quad (30)$$

We will call the point where the optical axis of the instrument intersects with the surface of Mars the trace of the optical axis which is defined by the vector $\bar{R}_\delta(x_\delta, y_\delta, z_\delta)$ (figure). The vector $\bar{R}_\delta$ will be determined from the condition that the line in the direction $\bar{l}_i^\circ$ intersects with the surface of Mars, which will be considered as an ellipsoid of revolution with equatorial radius R_δ and oblateness α_δ . Also, the condition $\bar{l}_i^\circ \cdot \bar{l}_\delta^\circ < 0$ must be satisfied.

1. If $\bar{l}_{i\delta}^\circ \neq 0$, we obtain the equation

$$A x_{\delta i k}^2 + B x_{\delta i k} + C = 0,$$

for the coordinates $k=1, 2$, where

$$\begin{aligned} A &= (1 - \alpha_\delta)^2 (1 - \bar{l}_{i\delta}^{\circ 2}) + \bar{l}_{i\delta}^{\circ 2}, \\ B &= 2[(1 - \alpha_\delta)^2 \bar{l}_{i\delta}^\circ (\bar{l}_{i\delta}^\circ y_{\delta\delta} - \bar{l}_{i\delta}^\circ x_{\delta\delta}) + \bar{l}_{i\delta}^\circ (\bar{l}_{i\delta}^\circ z_{\delta\delta} - \bar{l}_{i\delta}^\circ x_{\delta\delta})], \\ C &= (1 - \alpha_\delta)^2 (\bar{l}_{i\delta}^\circ y_{\delta\delta} - \bar{l}_{i\delta}^\circ x_{\delta\delta})^2 + (\bar{l}_{i\delta}^\circ z_{\delta\delta} - \bar{l}_{i\delta}^\circ x_{\delta\delta})^2 - R_\delta^2 (1 - \alpha_\delta)^2 \bar{l}_{i\delta}^{\circ 2}, \end{aligned} \quad (31)$$

$\bar{l}_{i\delta}^\circ, \bar{l}_{i\delta}^\circ, \bar{l}_{i\delta}^\circ$ are the direction cosines of the vector $\bar{l}_i^\circ$,
 $x_{\delta\delta}, y_{\delta\delta}, z_{\delta\delta}$ are the areocentric coordinates of the spacecraft.

The coordinates $y_{\delta i k}$ and $z_{\delta i k}$ are expressed in terms of $x_{\delta i k}$.

$$\begin{aligned} y_{\delta i k} &= \frac{\bar{l}_{i\delta}^\circ}{\bar{l}_{i\delta}^\circ} (x_{\delta i k} - x_{\delta\delta}) + y_{\delta\delta}, \\ z_{\delta i k} &= \frac{\bar{l}_{i\delta}^\circ}{\bar{l}_{i\delta}^\circ} (x_{\delta i k} - x_{\delta\delta}) + z_{\delta\delta}. \end{aligned} \quad (32)$$

Using the condition

$$l_k = |\bar{R}_{\sigma k} - \bar{R}_{\sigma \Delta}| = \min |\bar{l}_{ik}| \quad (k=1,2). \quad (33)$$

the vector $\bar{R}_{\sigma k}$ is determined uniquely.

2. If $l_{ik}^{\circ} \neq 0$, clearly $\lambda_{\sigma k} = \lambda_{\sigma \Delta}$ and to determine $y_{\sigma k}$ we obtain /22
an equation which is analogous to Eq. (31) ($k = 1, 2$):

$$A_1 y_{\sigma k}^2 + B_1 y_{\sigma k} + C_1 = 0 \quad (34)$$

where

$$A_1 = (1 - \alpha_{\sigma})^2 l_{i\sigma}^{\circ 2} + l_{i2\sigma}^{\circ 2}$$

$$B_1 = 2l_{i2\sigma}^{\circ} (l_{i\sigma}^{\circ} z_{\sigma \Delta} - l_{i2\sigma}^{\circ} y_{\sigma \Delta})$$

$$C_1 = (1 - \alpha_{\sigma})^2 l_{i\sigma}^{\circ 2} x_{\sigma \Delta}^2 + (l_{i\sigma}^{\circ} z_{\sigma \Delta} - l_{i2\sigma}^{\circ} y_{\sigma \Delta})^2 - R_{3\sigma}^2 (1 - \alpha_{\sigma})^2 l_{i\sigma}^{\circ 2}$$

The coordinate $z_{\sigma k}$ is expressed in terms of $y_{\sigma k}$:

$$z_{\sigma k} = \frac{l_{i2\sigma}^{\circ}}{l_{i\sigma}^{\circ}} - (y_{\sigma k} - y_{\sigma \Delta}) + z_{\sigma \Delta}$$

3. Finally, if $l_{ik}^{\circ} = l_{i\sigma}^{\circ} = 0$, we obtain

$$\lambda_{\sigma k} = \lambda_{\sigma \Delta}$$

$$y_{\sigma k} = y_{\sigma \Delta}$$

$$z_{\sigma k}^2 + (1 - \alpha_{\sigma})^2 [R_{3\sigma}^2 - (x_{\sigma \Delta}^2 + y_{\sigma \Delta}^2)] = 0. \quad (35)$$

The nonuniqueness of Eqs. (34) and (35) is eliminated through condition (33).

In the case when Eqs. (31), (34), (35) do not have real roots, there is no trace on the surface of Mars. The value of the vector can be used to determine the areodesic latitude $\bar{R}_{\sigma k}$ and $B_{\sigma k}$ longitude $L_{\sigma k}$ of the trace from the formulas:

$$\cos \alpha_{\sigma_i} = \frac{x_{\sigma_i}}{\sqrt{x_{\sigma_i}^2 + y_{\sigma_i}^2}}; \quad \sin \alpha_{\sigma_i} = \frac{y_{\sigma_i}}{\sqrt{x_{\sigma_i}^2 + y_{\sigma_i}^2}}; \quad \sin \delta_{\sigma_i} = \frac{z_{\sigma_i}}{|\bar{R}_{\sigma_i}|};$$

$$B_{\sigma_i} = \arctg \frac{1}{1 - \alpha_{\sigma_i}} \tan \delta_{\sigma_i}, \quad L_{\sigma_i} = \alpha_{\sigma_i} - S_{\sigma_i},$$
(36)

where α_{σ_i} and δ_{σ_i} are the right ascension and declination of the 23 trace (see figure) and S_{σ_i} is the sidereal time at the zero meridian of Mars at the given instant.

For the time derivatives $\dot{l}_i, \dot{R}_{\sigma_i}, \dot{l}_{\sigma_i}$, we obtain the formulas

$$\dot{l}_i = \frac{1}{l_i} \left[(\bar{R}_{\sigma_i} - \bar{R}_{\sigma_i}) (\dot{\bar{R}}_{\sigma_i} - \bar{V}_{\sigma_i}) \right];$$

$$\dot{B}_{\sigma_i} = \frac{(1 - \alpha_{\sigma_i}) [\dot{x}_{\sigma_i}^2 + \dot{y}_{\sigma_i}^2] \dot{z}_{\sigma_i} - z_{\sigma_i} (\dot{y}_{\sigma_i} \dot{x}_{\sigma_i} + \dot{x}_{\sigma_i} \dot{y}_{\sigma_i})}{\sqrt{x_{\sigma_i}^2 + y_{\sigma_i}^2} [(1 - \alpha_{\sigma_i}) (\dot{x}_{\sigma_i}^2 + \dot{y}_{\sigma_i}^2) + \dot{z}_{\sigma_i}^2]};$$

$$\dot{l}_{\sigma_i} = \frac{x_{\sigma_i} \dot{y}_{\sigma_i} - y_{\sigma_i} \dot{x}_{\sigma_i}}{\dot{x}_{\sigma_i}^2 + \dot{y}_{\sigma_i}^2} - \tilde{\omega}_{\sigma_i} \text{ , rate}$$
(37)

where $\dot{\bar{R}}_{\sigma_i} = \{\dot{x}_{\sigma_i}, \dot{y}_{\sigma_i}, \dot{z}_{\sigma_i}\},$
 $\bar{V}_{\sigma_i} = \{V_{x\sigma_i}, V_{y\sigma_i}, V_{z\sigma_i}\}.$

$\tilde{\omega}_{\sigma_i}$ is the angular velocity with which Mars rotates.

The vector $\bar{V}_{\sigma_i}$ is obtained by integrating the equations of motion of the spacecraft (1) and to find $\dot{\bar{R}}_{\sigma_i}$, we differentiate expressions (31), (32), or (34) or (35) with respect to t with the condition $\dot{\bar{r}}_{\sigma_i} = \text{const}$. For example, in case 1 we have:

$$\dot{x}_{\sigma_i K} = \frac{-\dot{B} \sqrt{B^2 - 4AC} \pm (B\dot{B} - 2AC)}{2A \sqrt{B^2 - 4AC}} \quad (K=1,2),$$
(38)

where

$$\dot{A} = 0;$$

$$\dot{B} = 2[(1 - \alpha_{\sigma_i})^2 \dot{r}_{\sigma_i} \dot{r}_{\sigma_i} V_{x\sigma_i} - \dot{r}_{\sigma_i} \dot{r}_{\sigma_i} V_{x\sigma_i}] \cdot \dot{r}_{\sigma_i} \dot{r}_{\sigma_i} V_{z\sigma_i} - \dot{r}_{\sigma_i} \dot{r}_{\sigma_i} V_{x\sigma_i}],$$

$$\dot{C} = 2[(1 - \alpha_{\sigma_i})^2 (\dot{r}_{\sigma_i} V_{x\sigma_i} - \dot{r}_{\sigma_i} V_{x\sigma_i}) (\dot{r}_{\sigma_i} V_{x\sigma_i} - \dot{r}_{\sigma_i} V_{x\sigma_i}) \cdot (\dot{r}_{\sigma_i} V_{z\sigma_i} - \dot{r}_{\sigma_i} V_{z\sigma_i}) (\dot{r}_{\sigma_i} V_{x\sigma_i} - \dot{r}_{\sigma_i} V_{x\sigma_i})].$$

Condition (33) determines the choice of sign in (38). For the components $\dot{\gamma}_{\delta_i}$ and $\dot{z}_{\delta_i}$, we obtain:

$$\left. \begin{aligned} \dot{\gamma}_{\delta_i} &= \frac{\ell_{ix\delta}^{\circ} V_{y\delta} - \ell_{iy\delta}^{\circ} V_{x\delta} + \ell_{iz\delta}^{\circ} \dot{x}_{\delta_i}}{\ell_{ix\delta}^{\circ}} \\ \dot{z}_{\delta_i} &= \frac{\ell_{ix\delta}^{\circ} V_{z\delta} - \ell_{iz\delta}^{\circ} V_{x\delta} + \ell_{iz\delta}^{\circ} \dot{x}_{\delta_i}}{\ell_{ix\delta}^{\circ}} \end{aligned} \right\} \quad (39)$$

We introduce into the discussion a local coordinate system /24 X_1, Y_1, Z_1 (see figure) referred to the trace on the surface of Mars. The Z_1 axis is normal to the surface of Mars, the X_1 axis is tangent to the south meridian, and the Y_1 axis makes the system a right-handed coordinate system. In this coordinate system, the azimuth A and elevation angle η_1 in the direction $-\vec{\ell}_i^{\circ}$ can be determined from the formulas:

$$\left. \begin{aligned} \cos A_i &= -\frac{\ell_{ix_1}^{\circ}}{\sqrt{\ell_{ix_1}^{\circ 2} + \ell_{iy_1}^{\circ 2}}}; \quad \sin A_i = -\frac{\ell_{iy_1}^{\circ}}{\sqrt{\ell_{ix_1}^{\circ 2} + \ell_{iy_1}^{\circ 2}}}; \quad \sin h_i = -\ell_{iz_1}^{\circ} \end{aligned} \right\} \quad (40)$$

where $\ell_{ix_1}^{\circ}, \ell_{iy_1}^{\circ}, \ell_{iz_1}^{\circ}$ are the coordinates of the vector $\vec{\ell}_i^{\circ}$ in the X_1, Y_1, Z_1 system, which are defined by the expressions

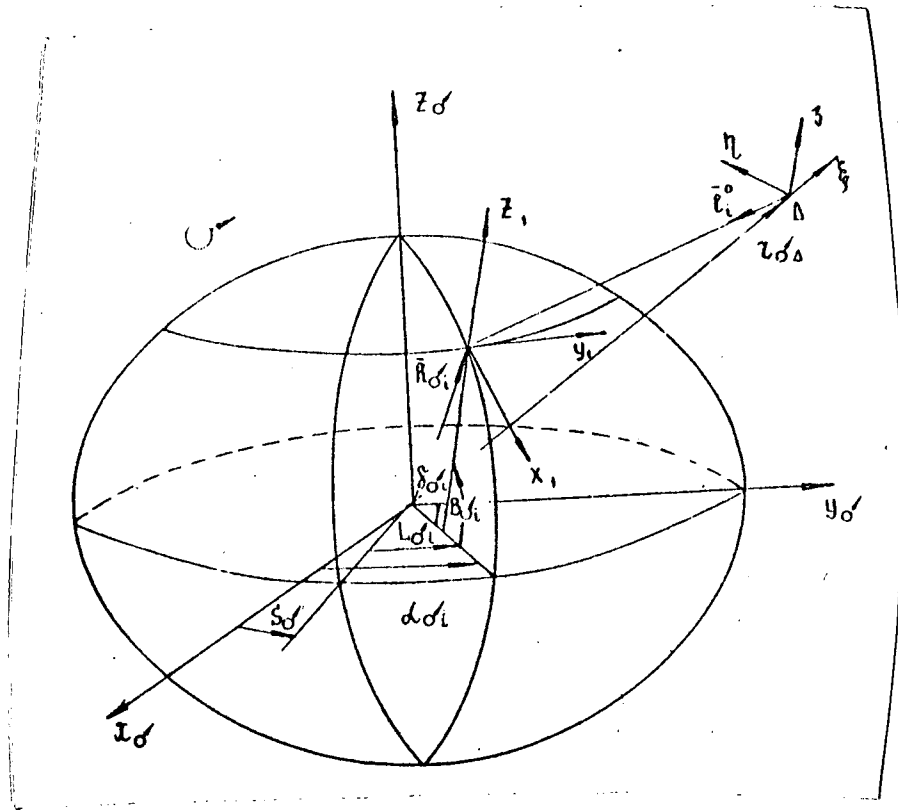
$$\left. \begin{aligned} \left\| \begin{array}{c} \ell_{ix_1}^{\circ} \\ \ell_{iy_1}^{\circ} \\ \ell_{iz_1}^{\circ} \end{array} \right\| &= \left\| \begin{array}{ccc} \sin \theta_{\delta_i} \cos L_{\delta_i} & \sin \theta_{\delta_i} \sin L_{\delta_i} & -\cos \theta_{\delta_i} \\ -\sin L_{\delta_i} & \cos L_{\delta_i} & 0 \\ \cos \theta_{\delta_i} \cos L_{\delta_i} & \cos \theta_{\delta_i} \sin L_{\delta_i} & \sin \theta_{\delta_i} \end{array} \right\| \cdot \left\| \begin{array}{c} \ell_{ix\delta}^{\circ} \\ \ell_{iy\delta}^{\circ} \\ \ell_{iz\delta}^{\circ} \end{array} \right\| \end{aligned} \right\} \quad (41)$$

V. Astrodynamic Constants Used in the Calculations

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We first note that the values of the astrodynamic constants, especially those referred to Mars, can vary from flight to flight, since the numerical values of some of these constants are known only very approximately. Below we give the main constants which were used in work with the "Mars-1" and "Mars-2" spacecraft in the years 1971 through 1972.

$\mu_{\oplus} = 398\,600.9 \text{ km}^3/\text{sec}^2$ Earth gravitational parameter
 $R_{\oplus} = 6\,378.142 \text{ km}$ Earth equatorial radius
 $C_{20\oplus} = -0.001082639$ coefficient of the zonal harmonic
of Earth gravitation
 $\mu_{\text{Moon}} = 4\,902.84 \text{ km}^3/\text{sec}^2$ gravitational parameters of the Moon,
 $\mu_{\odot} = 132\,712\,517\,000 \text{ km}^3/\text{sec}^2$ the Sun,
 $\mu_{\text{Mars}} = 42\,828.48 \text{ km}^3/\text{sec}^2$ Mars
 $\mu_{\text{Jup}} = 126\,498\,000 \text{ km}^3/\text{sec}^2$ and Jupiter
 $\chi_{\Delta} = 637\,019.962 \text{ km}^3/\text{sec}^2$ coefficient of light radiation
 $\tilde{\omega}_{\text{Mars}} = 350.891962 \text{ }^{\circ}/\text{eph. day}$ angular velocity with which Mars
rotates
 $\Delta T^{(a)} = 0.00047$ ephemeris correction in 1971
 $R_{\text{Mars}} = 3\,393.4 \text{ km}$ equatorial radius of Mars
 $\epsilon_{\text{Mars}} = 0.00525$ oblateness of Mars
 $C_{20\text{Mars}} = -0.00197$ coefficient of zonal harmonic of
Mars gravitation
 $a_{\text{AU}} = 149\,600\,000 \text{ km}$ astronomical unit
 $\sin \mu_{\text{L}} = 0.01659251$ sine of the horizontal lunar
parallax constant



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